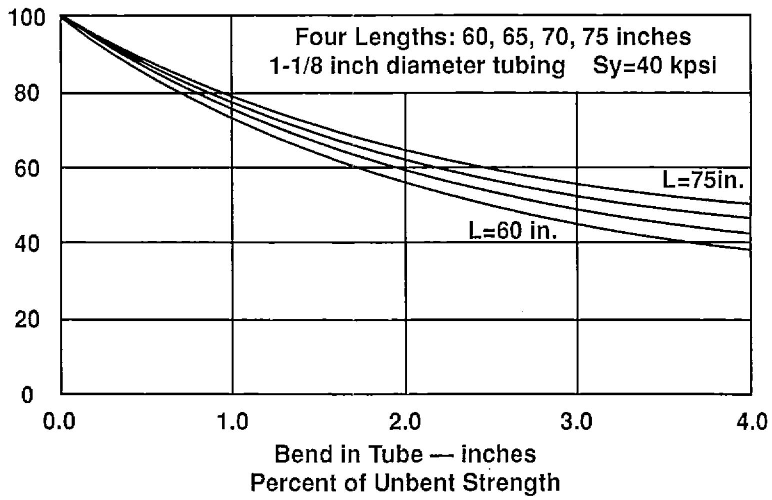
Column Failure of Bent Tubes

by Richard Cobb



It has often been noted that a column member that is not perfectly straight is significantly less strong than one that is. The purpose of this article is to illustrate this fact with concrete values.

A classical "column" is a member loaded axially in compression (such as downtubes). Most theoretical calculations are based on columns that are perfectly straight and of constant cross section over the entire length. There are two classes of column calculations, generally known as the "Euler" (pronounced "oiler") and "J.B. Johnson" equations. The J.B. Johnson equations apply to short, thick columns (such as one with the dimensions of a beer can). These members fail by yielding (plastic deformation) of the column material, and are of no interest here. The Euler equations apply to long and slender columns, such as our downtubes. The remainder of this discussion will be limited to this class of columns. An Euler column's buckling load *is not dependent on the strength of the material.* The only material property that is important is the modulus of elasticity, which relates loads and deflections. This is a relative constant for a given material, independent of the strength and varying only slightly with production method and alloy ingredients. That is, all aluminums have one modulus of elasticity, all steels have another (steel is almost three times "stiffer" than aluminum).

The underlying theory of an Euler column treats failure as a stability problem. Whereas we can bend a beam (such as a leading edge) and have some indication both of the load being applied and the nearness of failure (based on the severity of the bend), columns offer no such visual warnings. A heavily-loaded column appears no different than a lightly loaded one. As we increase the load, however, we reach a point where the column becomes unstable. Collapse is sudden and with no warning.

The following assumptions are inherent in the Euler equations: the column is perfectly straight; the load is applied at the ends of the column and is perfectly centered; the load acts along the center line of the column; and there are no other loads (such as bending) applied. We commonly violate these assumptions in two ways: we sometimes apply bending loads (such as during a landing flare) and we sometimes fly with bent tubes. The calculations for failure under either of these conditions is considerably more complicated than under the Euler assumptions. Since our landing flares are close to the ground and at (hopefully!) low airspeeds (and therefore lower loads), those calculations won't be covered here.

The "yield" strength of a material (S_y) is defined as the stress applied when permanent deformation occurs. (Your bent tube is the result of bending stresses which exceeded the yield strength at some earlier time). While material yield strength has no place in the calculation of buckling load of an Euler column, it does come into the calculations when the column is bent. The maximum stress occurs on the concave side of that part of the bent column which is farthest away from the line drawn through the ends of the column. When this stress exceeds the yield strength the material in this region plastically deforms ("yields"). This, of course, causes a larger bending moment and the column collapses. The equation for the maximum stress (σ) in a curved column[1] is:

$$\sigma = \frac{P}{A}\left(1 + \frac{dc}{r^2}\,\frac{8EA}{P(L/r)^2}\left\{\sec\left[\frac{P}{4EA}\left(\frac{L}{r}\right)^2\right]^{1/2} - 1\right\}\right)$$

This equation is presented mainly to show the nature of the computation. P is the load applied axially at the ends of the column, A is the cross section area of the column, d is the maximum initial deflection (bend), c (in our case) is the outer radius of the tube, r is the radius of gyration (a property of a cross sectional which is related to the area moment of inertia), L is the length of the column, and E is the material modulus of elastic-

DISCLAIMER!

The purpose of this article is NOT to encourage anyone to fly with bent downtubes by trying to guess "how much they can get away with." Remember: THE FAILURE OF A COLUMN MEMBER IS SUDDEN AND CATASTROPHIC—YOU HAVE NO WARNING OF IMPENDING FAILURE! Also: These are theoretical calculations and should be taken as such. They were not backed up by experimental verification and may not accurately predict actual buckling loads.

ity. The difficulty of this computation is to find the load P which causes the maximum stress to equal the yield strength (Sy) of the material. This is not easy because P appears three times in a non-linear equation. The only way to get the answer is to use an iterative "numerical solution" technique. This is a fancy way of saying that you use a computer to guess at values of P until the values on both sides of the "=" sign are pretty close.

If you don't have a solid mechanics background you're probably close to asleep by now. Let's get on to the more interesting results.

TABLE 1

Euler Column Buckling Loads (pounds)
(1-1/8" outside diameter aluminum tube)

Wall Thickness	Length (inches)			
	60	65	70	75
0.058"	784	668	576	501
0.095"	1161	989	853	743

Table 1 shows the Euler buckling loads for downtubes made of 1-1/8 inch diameter aluminum tubing with different lengths and thicknesses. Figure 1 shows the percent of Euler strength remaining in the tubes when there is an initial curvature. These curves apply to any thickness tube with 1-1/8 inch outer diameter, a yield strength of 40,000 psi and the same modulus of elasticity as aluminum: 10.3×10^6 psi (depending on the reference text—I have seen this value range from 10.0 to nearly 11).

Note how rapidly the curve drops at first. Even a one-inch bend in a tube can reduce strength by 20 to 30 percent! (A one-inch bend means that some part of the tube lies an inch from where it would be if the tube were straight.) Also, be careful about how you interpret the graph. It would appear that a 70-inch-long tube with a one-inch bend is stronger that a 60-inch tube with the same bend. Remember that these are only percentages of the Euler buckling load. Table 1 shows that the 70-inch tube is much weaker to begin with than the 60-inch tube. So even though it has a higher percentage of its unbent strength, the bent 70-inch tube will still fail under a smaller load than the bent 60-inch tube.

A note about material strength and bent columns: As noted earlier, an unbent column calculation does not consider yield strength for buckling loads while a bent column calculation does. So how much safer is a higher-strength alloy (say 7075 versus 6061)? The answer is not all that much. For a two- to three-inch bend, *doubling* the yield strength only increases the buckling load by about 30 percent. (For these calculations a yield strength of 40,000 psi was used—a "typical" value for 6061-T6.)

Finally, this information does us no good without some kind of idea of what in-flight loads are. Doing some crude calculations I came up with a "ball park" (or should I say "LZ"?) figure of about a 100-pound compression load on each downtube in normal (single-G) flight for my Sensor. I suspect that this would apply to a lot of other gliders as well.

What about downtubes with inner sleeves? Well, remember that all of these calculations are based on the assumption of constant cross section. Since most sleeves are shorter than the outer tube this assumption is violated. For practical purposes, however, we can probably get a rough estimate by using a thickness equal to the two individual thicknesses added. But for this to even come close I suspect that the inner tube should be a tight fit and not much shorter than the outer tube.

For those who wish to pursue these calculations and have access to an IBM PC-compatible computer, I have written a program which carries out the calculation rapidly. It is written in Turbo Pascal and will also handle non-tubular columns if you know such things as the cross section moment of inertia and the cross section area. ■

The author started hang gliding in 1981 and is currently an Advanced-rated pilot and Instructor/Observer. He recently completed a Ph.D. in mechanical engineering at Virginia Polytechnic Institute and State University.

The program mentioned is available for the cost of the 5-1/4 inch disk and postage ($5.00): Richard Cobb, 620 W. Foster Ave., State College, PA 16801.—Ed.

REFERENCE

1. Roark, Raymond J., and Young, Warren C., *Formulas for Stress and Strain*, 5th edition, McGraw-Hill, New York, 1975, page 433.